Post-inflationary behavior of adiabatic perturbations and tensor-to-scalar ratio

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We explain why it is so difficult and perhaps even impossible to increase the cosmological tensor-to-scalar perturbation ratio during the post-inflationary evolution of the universe. Nevertheless, contrary to some recent claims, tensor perturbations can be relatively large in the simplest inflationary models which do not violate any rules of modern quantum field theory.

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I. INTRODUCTION

Tensor perturbations (gravitational waves) [1] produced during inflation [2, 3, 4] may serve as an important tool helping us to discriminate among different types of inflationary models. These perturbations typically give a much smaller contribution to the CMB anisotropy than the inflationary adiabatic scalar perturbations [5, 6, 7]. Nevertheless, in the simplest versions of chaotic inflation models with potentials $\sim \phi^n$ [8] the amplitude of tensor perturbations can be large enough to be detected. Meanwhile in most of the other models, such as new inflation [9] and hybrid inflation [10], tensor perturbations typically are too small [3]. Therefore a discovery of the B-mode of the CMB polarization, which is related to tensor metric perturbations, would be a strong argument in favor of the simplest versions of chaotic inflation, whereas the absence of the B-mode would rule out the simplest versions of chaotic inflation without helping much in distinguishing between many other versions of inflationary theory.

Recently it was argued, however, that the contribution of tensor perturbations to the CMB anisotropy can be much greater than expected. One may consider the models where inflation occurs at very high energy scales, which increases the amplitude of scalar and tensor perturbations. Then one may try to find a way to significantly suppress the amplitude of scalar perturbations due to some post-inflationary evolution. This would boost up the relative amplitude of tensor perturbations and make them detectable in a broad class of inflationary models [11, 12].

The basic idea of Refs. [11, 12] can be explained as follows. Suppose, for example, we have a hot post-inflationary universe with energy density $\rho \sim T^4$, and with density perturbations $\delta\rho \sim 4T^3\delta T$. Now let us assume, for example, that some homogeneous scalar field begins to dominate the evolution of the universe, and then it decays. If the field decay occurs simultaneously in all parts of the universe, it increases the total energy density of radiation without increasing the amplitude of its perturbations. As a result, the ratio $\delta\rho/\rho$ decreases.

The problem with this idea is that such a decay cannot occur in all parts of the universe simultaneously. To be more precise, one may work in the gauge where the spatial scalar curvature of the universe vanishes, called the flat slicing. In this gauge, the Hubble constant and density ρ are functions of both space and time, $\rho = \rho(t, x^i)$, $H = H(t, x^i)$, but they are related to each other by the standard flat space equation $H^2 = \rho/3$, in units $M_p = 1$. Scalar perturbations of the metric in this gauge are directly related to the non-simultaneous end of inflation in different parts of the universe, due to quantum fluctuations of the inflaton field. One may think about different parts of the universe of a superhorizon size as of separate flat universes which started their post-inflationary expansion at slightly different moments of time determined by $\rho(t + \delta t, x^i) = \rho_e$, where ρ_e is the energy density at the end of inflation. The subsequent dynamics of all physical processes in the universe on the superhorizon scale will be determined not by the global time t, but by the local values of density of matter, which will be a function of $t + \delta t$.

A more rigorous, mathematical description of the above is possible in terms of the δN formalism, in which the amplitude of the adiabatic perturbation generated during inflation is given by the difference in the number of e-foldings relative to the background from the time of horizon crossing during inflation on the flat hypersurface to the end time of inflation which occurs on a comoving (or uniform density) hypersurface [13, 14, 15, 16, 17, 18].

Consider for example any other scalar field σ. Suppose that this field decays after it becomes dominating the energy density of the universe. This situation is very similar to the one encountered in the curvaton scenario, but now we will assume, following [11, 12], that the fluctuations generated in this field during inflation are negligibly small, so that they do not induce any new adiabatic perturbations after the field σ decays. The main assumption made in [11, 12] was that this field begins oscillating or decays at the same time t in all parts of the universe. But the field σ does not "know" anything about the global

cosmological time t, it depends only on the local energy density of the universe, local temperature, etc.

This means, for example, that a massive field σ with an effective potential $m_\sigma^2\sigma^2/2$ will start oscillating not at some given time t, but at the time $t+\delta t$ when the local value of Hubble constant $H(t+\delta t)$ will become smaller than m_σ. At that moment, energy density $m_\sigma^2\sigma^2/2$ of the homogeneous field σ everywhere in the universe will constitute the same fraction of the local energy density of mater given by $3H^2(t+\delta t)$, and, consequently, the same fraction of the energy density of radiation. As a result, the motion of the field σ, its density, and its subsequent decay will be synchronized in the same way as all other matter in the universe. This is the main reason why its decay cannot reduce the amplitude of adiabatic perturbations; they will be simply redistributed from one type of matter to another.

For the particular models studied in [11, 12] this negative conclusion follows directly from the standard theory of inflationary perturbations in the theories involving only one scalar field [5, 6, 7]. Indeed, the second scalar field discussed in these models behaved essentially like a decaying fluid; no effects related to inflationary quantum fluctuations induced in the second field were considered in [11, 12]; in this respect see also comments in Ref. [19].

The authors of Refs. [11, 12] also realized that the original models described in their papers did not provide damping of adiabatic perturbations. Still one may wonder whether one could propose a different, more sophisticated mechanism to suppress superhorizon inflationary perturbations. In particular, instead of the possibility to "dilute" the original adiabatic perturbations as suggested in [11, 12], one could speculate about the possibility to generate isocurvature perturbations [20], convert them to adiabatic perturbations by the curvaton mechanism [21, 22, 23, 24] (or use the mechanism proposed in [25]), and then fine-tune the amplitude and phases of these perturbations to cancel the original adiabatic perturbations. In other words, instead of damping the adiabatic perturbations, one could try to cancel them by perturbations of a different type.

In this paper we will argue that there is no natural way to suppress the original adiabatic perturbations. In particular, adiabatic perturbations generated by the curvaton mechanism are produced from quantum fluctuations of the curvaton field, which are generated independently of the quantum fluctuations of the inflaton field. Therefore even if one fine-tunes the *amplitude* of the curvaton perturbations, they cannot have the same *phase* as the initial adiabatic perturbations, which would be necessary for their cancellation.

Our results provide an additional demonstration of robustness of inflationary predictions. On the other hand, one could worry that if it is impossible to boost the tensor-to-scalar ratio, then our chances to detect tensor perturbations are very slim. This point of view was expressed in [11, 26], where it was argued that it is difficult to construct a satisfactory model of inflation predicting detectable tensor perturbations. In Section III we explain why we are more optimistic in this respect.

II. EVOLUTION OF THE SUPERHORIZON SCALAR PERTURBATIONS

The theory of cosmological perturbations in inflationary theories involving one scalar field, the inflaton, is well-developed, see, e.g., [2, 3, 4, 6, 7]. There is no room for speculation there, and the conclusion about the ratio of the gravitational waves to adiabatic perturbations is robust. Namely, the amplitude of tensor perturbations is suppressed compared to the amplitude of scalar perturbations by a factor of order $(1+p/\varepsilon) \ll 1$, where the ratio of the effective pressure p to the energy density ε is estimated when the mode with the corresponding comoving wavenumber crosses the horizon during inflation. Therefore to find out whether the ratio of the tensor and scalar perturbations can substantially grow due to some post-inflationary physical processes one must consider multi-component models, where different scalar fields play important roles at different stages of evolution. For instance one could consider the situation when after inflation (due to the inflaton), another scalar field (for example, curvaton), which was subdominant during inflation, takes over and begins to dominate. Assuming that this second scalar field was initially distributed nearly homogeneously, one may wonder whether the inflationary metric fluctuations can be suppressed after this initially homogeneous scalar begins to dominate. We will argue below that the original inflationary perturbations will be generically transferred to the second scalar field and hence the resulting fluctuations will be at least as large as the original inflationary fluctuations.

A. Isentropic fluids

In order to make our conclusions intuitively clear, we begin with the case of isentropic ideal hydrodynamical fluids, where one can find an explicit solution for superhorizon modes even in multicomponent media. It is very instructive to find out how the adiabatic perturbations generated in one of these fluids become transferred to other liquids even if they interact only gravitationally.

If different ideal fluids interact only gravitationally then the conservation law

$$T^{\alpha}_{\beta;\alpha} = 0 \tag{1}$$

is valid for every component separately. Taking into account that the off-diagonal spatial components of the energy-momentum tensor vanish the metric of a flat universe with small scalar perturbations can be written as

(see, for example, [4])

$$ds^2 = a^2(\eta)\left[(1+2\Phi)\,d\eta^2 - (1-2\Phi)\,\delta_{ik}dx^i dx^k\right]. \quad (2)$$

This is called the longitudinal or Newton gauge. In what follows, perturbation variables are those defined in this gauge unless otherwise stated. Then for *each* fluid the linearized conservation law $T^{\alpha}_{0;\alpha} = 0$ takes the following form:

$$\delta\varepsilon_J' + 3\mathcal{H}(\delta\varepsilon_J + \delta p_J) - 3(\varepsilon_J + p_J)\Phi' + a(\varepsilon_J + p_J)u^i_{,i} = 0. \quad (3)$$

Here $\mathcal{H} = a'/a$, the prime denotes the derivative with respect to the conformal time η, and J numerates different fluids. On the superhorizon scales, the last term in Eq. (3) can be neglected. Using equation

$$\varepsilon_J' = -3\mathcal{H}(\varepsilon_J + p_J), \quad (4)$$

which is also valid for each fluid separately, we can rewrite (3) as

$$\left[\frac{\delta\varepsilon_J}{\varepsilon_J + p_J} - 3\Phi\right]' = \frac{\varepsilon_J'\delta p_J - p_J'\delta\varepsilon_J}{(\varepsilon_J + p_J)^2}. \quad (5)$$

Introducing standard notations

$$\zeta_J = \Phi - \frac{\delta\varepsilon_J}{3(\varepsilon_J + p_J)} = \Phi + \mathcal{H}\frac{\delta\varepsilon_J}{\varepsilon_J'},$$
$$\delta p_J^{\mathrm{nad}} = \delta p_J - \frac{p_J'}{\varepsilon_J'}\delta\varepsilon_J, \quad (6)$$

where ζ_J describes the curvature perturbation on slices of homogeneous density for each fluid component, one can represent Eq. (5) in the following form:

$$\zeta_J' = \frac{\mathcal{H}}{\varepsilon_J + p_J}\delta p_J^{\mathrm{nad}}. \quad (7)$$

Note that in deriving (7) we did not use any Einstein equations for perturbations.

In the case of isentropic fluids $p_J = p_J(\varepsilon_J)$ we have

$$\delta p_J^{\mathrm{nad}} = \delta p_J - \frac{p_J'}{\varepsilon_J'}\delta\varepsilon_J = 0 \quad (8)$$

for each component and

$$\zeta_J' = 0. \quad (9)$$

Hence ζ_J is conserved for each fluid separately on superhorizon scales. We would like to point out that when we skipped the last term in (3) we have lost the decaying mode of the scalar perturbations, which anyway soon becomes negligible.

Let us introduce the "master ζ"

$$\zeta \equiv \Phi + \frac{2}{3}\frac{\Phi' + \mathcal{H}\Phi}{(1 + p/\varepsilon)\mathcal{H}}, \quad (10)$$

where p and ε are the *total* pressure and the energy density respectively. Note that this variable is equal to minus the curvature perturbation on comoving slices, often denoted by $\mathcal{R}_c$, i.e., $\zeta = -\mathcal{R}_c$ [14]. Although the curvature perturbation on comoving slices and the curvature perturbation on homogeneous density slices are not identical, they coincide on superhorizon scales. Using the $(0,0)$-component of Einstein equations

$$\overset{(3)}{\Delta}\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = 4\pi G a^2\left(\sum \delta\varepsilon_J\right), \quad (11)$$

where one can neglect the first term for the long-wavelength perturbations, we find

$$\zeta = \frac{1}{(\varepsilon + p)}\left(\sum_J (\varepsilon_J + p_J)\zeta_J\right). \quad (12)$$

After inflation, when $\rho + p \sim O(\rho)$, the value of ζ is of order the gravitational potential, that is, $\Phi = O(1)\times\zeta$. In distinction from ζ_J, the master ζ is not in general conserved. However, irrespective of what happens with the perturbations at some intermediate time scale, Eq. (12) implies that at some final moment of time η_f, when the $J = F$ fluid dominates,

$$\zeta(\eta_f) \simeq \zeta_F(\eta_f) = \zeta_F(\eta_i), \quad (13)$$

where the latter equality follows from the conservation of ζ_J. At the initial moment

$$\zeta_F(\eta_i) = \left(\Phi - \frac{\delta\varepsilon_F}{3(\varepsilon_F + p_F)}\right)_{\eta_i}. \quad (14)$$

Here the gravitational potential $\Phi(\eta_i)$ is mostly due to the fluid component $J = I$, which dominates at η_i and is equal to (see, for example, [4])

$$\Phi(\eta_i) \simeq -\frac{1}{2}\frac{\delta\varepsilon_I}{\varepsilon_I}(\eta_i). \quad (15)$$

It follows from (13) and (14) that the final value of $\Phi(\eta_f) = O(1)\times\zeta(\eta_f)$ is in general never smaller than $\Phi(\eta_i)$. Even if the component which finally dominates was initially distributed homogeneously, i.e. $\delta\varepsilon_F(\eta_i) = 0$, we obtain, nevertheless, $\Phi(\eta_f) \sim \Phi(\eta_i)$. Only in the exceptional case when

$$\Phi(\eta_i) \simeq -\frac{1}{2}\frac{\delta\varepsilon_I}{\varepsilon_I}(\eta_i) = \frac{\delta\varepsilon_F}{3(\varepsilon_F + p_F)}(\eta_i), \quad (16)$$

the resulting metric perturbations can be strongly suppressed compared to their initial values. However, it is impossible to realize in a natural way the condition (16) because it requires the coherent correlated distribution of the fluids at all wavelengths.

B. Scalar fields

Unfortunately the results obtained above cannot be directly reformulated for several scalar fields. In fact, there

is no analog of separately conserved ζ_J in this case even if the scalar fields interact only gravitationally. However, one can argue in a similar way that the cancellation of the dominant adiabatic mode is also quite unlikely in this case. We first consider for simplicity several noninteracting scalar fields φ_J with potentials $V_J(\varphi_J)$. Then their contributions to the total energy density and pressure are given by

$$\begin{aligned}\varepsilon_J &= \frac{1}{2} g^{\alpha\beta}\varphi_{J,\alpha}\varphi_{J,\beta} + V_J(\varphi_J), \\ p_J &= \frac{1}{2} g^{\alpha\beta}\varphi_{J,\alpha}\varphi_{J,\beta} - V_J(\varphi_J).\end{aligned}$$

For several scalar fields it is convenient to introduce

$$\zeta_J \equiv \Phi - \frac{\delta\varepsilon_J}{3(\varepsilon_J + p_J)} \times \frac{(1 - \delta p_J/\delta\varepsilon_J)}{(1 - p'_J/\varepsilon'_J)} = \Phi + \mathcal{H}\frac{\delta\varphi_J}{\varphi'_J} \quad (17)$$

instead of ζ_J defined in (6), which describes the curvature perturbation on slices comoving with each scalar field.[1] Using Einstein equations for perturbations one can easily verify that only for the case of a single scalar field the ζ_J defined in (6) and (17), are equal; otherwise they are different. The conservation equation (3) is still valid for each scalar field separately. However, in this case one cannot reduce it to a useful form similar to (7) with $\delta p_J^{\rm nad} = 0$. Therefore we consider instead the conservation of the total energy

$$\left(\sum_J \delta\varepsilon_J\right)' + 3\mathcal{H}\sum_J (\delta\varepsilon_J + \delta p_J) - 3\sum_J (\varepsilon_J + p_J)\,\Phi' = 0\,, \quad (18)$$

where we took the advantage of considering the superhorizon perturbations and skipped the spatial derivative terms. From (11), where we neglect $\overset{(3)}{\Delta}\Phi$, and the $(0,i)$-component of Einstein equations

$$(\Phi' + \mathcal{H}\Phi) = 4\pi G a^2 \sum_J (\varepsilon_J + p_J)\frac{\delta\varphi_J}{\varphi'_J}\,, \quad (19)$$

we find

$$\sum_J \delta\varepsilon_J = -3\mathcal{H}\sum_J (\varepsilon_J + p_J)\frac{\delta\varphi_J}{\varphi'_J}\,. \quad (20)$$

Substituting this expression into (18) and using the background equation of motion we finally obtain

$$\sum_J (\varepsilon_J + p_J)\,\zeta'_J = 0\,. \quad (21)$$

[1] Strictly speaking the ζ variable cannot be used to trace the behavior of scalar perturbations through the stage of oscillation of the scalar field because the next to leading k^2-corrections to the long-wavelength solution for ζ become infinite when φ' vanishes. The variable $v = a\varphi'\zeta/\mathcal{H} = a\,\delta\varphi_{flat}$, where $\delta\varphi_{flat}$ is the scalar field perturbation on flat slices, should be used instead (see, for example, [4]). However considering only the leading order behavior of ζ we ignore this subtlety here.

Taking the derivative of the master ζ (see Eqs. (10) and (12), which are valid for scalar fields) and using (21) one gets

$$\begin{aligned}\zeta' &= \frac{1}{(\varepsilon+p)^2}\sum_{J,K}(\varepsilon_J + p_J)'(\varepsilon_K + p_K)(\zeta_J - \zeta_K) \\ &\equiv \sum_{J,K} F_{JK}(\eta)(\zeta_J - \zeta_K)\,. \end{aligned} \quad (22)$$

One can verify that after the substitution $\varepsilon_J + p_J \to \varphi_J'^2/a^2$ the equations above remain valid for the interacting scalar fields. Eq. (22) implies the existence of the adiabatic mode for which

$$\zeta_1 = \zeta_2 = ... = \zeta = \text{const.}. \quad (23)$$

This equation is, of course, not enough to determine the behavior of all ζ_J for generic initial conditions. The variables ζ_J satisfy rather complicated coupled system of equations [27]. In distinction from the isentropic fluids, ζ_J for the scalar fields are not conserved separately. However, assuming that initially and finally the universe is dominated by one of the fields or by its decay products, one can make rather generic conclusions about the resulting value of $\Phi \sim \zeta$ based on (22).

If one makes the standard assumption that the inflaton field φ_I and its decay products dominate during and after inflation, one comes to the standard conclusion about the evolution of the amplitude of adiabatic perturbations. In this case, all post-inflationary processes on the superhorizon scale are synchronized by the original adiabatic perturbations of the metric and cannot be damped. Their final amplitude depends only on the final equation of state, but not on local processes at some intermediate time [5, 6, 7].

Now we will check whether this conclusion can be modified due to effects related to other scalar fields. We will assume that after inflation is over, the energy density released by the inflaton field decays faster than the energy density of some other (curvaton) scalar field φ_C and finally this field begins to dominate. The question is whether the metric perturbations, which are finally due to the curvaton field and its decay products, can be suppressed compared to the metric fluctuations originally generated by the inflaton.

Let us consider the mode with the comoving wavenumber k which crosses the horizon at the moment η_k during inflation. Integrating (22), we have for $\eta > \eta_k$

$$\zeta_k(\eta) = \zeta_{inf}(\eta_k) + \int_{\eta_k}^{\eta}(F_{IC} - F_{CI})(\zeta_I - \zeta_C)\,d\eta, \quad (24)$$

where $\zeta_{inf}(\eta_k)$ is the adiabatic mode due to the inflaton perturbations. It is clear from the definition in (22) that the functions F are non-negligible only when $\varepsilon_I + p_I \sim \varepsilon_C + p_C$. Only during this time interval ζ is not conserved. Let us assume that initially at $\eta = \eta_k$ there exists also

an isocurvature mode in addition to the adiabatic mode, so that

$$S_{IC}(\eta) \equiv \zeta_I - \zeta_C = f(\eta)\, S_{IC}(\eta_k)\,,$$

where $f(\eta)$ does not depend on k. Then at some late moment of time η_f when the curvaton dominates and ζ is conserved, we obtain

$$\zeta_k(\eta_f) = \zeta_{inf}(\eta_k) + \left(\int (F_{IC} - F_{CI})\, f(\eta)\, d\eta\right) S_{IC}(\eta_k)\,, \tag{25}$$

where the integral is some $k-$independent constant. It is clear that $\zeta_k(\eta_f)$ can be substantially reduced only if the second term in (25) compensates the first one with high accuracy. This means that the isocurvature mode should be initially highly correlated with the adiabatic mode, the situation which we believe is not only contrived but in fact impossible. Indeed, the fluctuations of the inflaton and the curvaton field occur independently. These fluctuations have quantum origin, they represent two independent degrees of freedom, and their phases are random. As a result, they cannot cancel each other. Instead of that, the square of the amplitude of a combination of these two types of oscillations will be given by the sum of the squares of the amplitudes of both types. In other words, the curvaton contribution can only increase the total amplitude of adiabatic perturbations. Generically the final amplitude is determined by the largest of the two terms in (25). If the first term dominates, which is the simplest and the most general possibility, then the final perturbations are mainly determined by the inflaton. Otherwise, when the second term dominates, they are due to the curvaton mechanism.

III. CAN WE DETECT TENSOR PERTURBATIONS?

Now we are going to analyze the argument of [11, 26] against the theories where the amplitude of tensor perturbations can be large. This argument was based on a correct observation that the amplitude of tensor perturbations can be large only in the theories where inflation occurs at $\phi \gtrsim M_p$ [26, 28]. But then the authors of [11, 15, 26] made an assumption that the generic expression for the effective potential can be cast in the form

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_n \lambda_n \frac{\phi^{4+n}}{{M_p}^n}\,, \tag{26}$$

and then they assumed that generically $\lambda_n = O(1)$, see e.g. Eq. (6) in [26] or Eq. (128) in [15]. This would imply that the behavior of $V(\phi)$ at $\phi > M_p$ is out of control.

Here we have written $M_p = 1/\sqrt{8\pi G}$ explicitly, to expose the implicit assumption made in [15, 26]. Why do we write M_p in the denominator, instead of $1000M_p$? An intuitive reason is that quantum gravity is nonrenormalizable, so one should introduce a cut-off at momenta $k \sim M_p$ [15, 26]. This is a reasonable assumption, but it does not imply the validity of Eq. (26). Indeed, the constant part of the scalar field by itself does not have any physical meaning. It appears in Feynman diagrams not directly, but only via its effective potential $V(\phi)$ and the masses of particles interacting with the scalar field ϕ. As a result, the terms induced by quantum gravity instead of the dangerous factors $\frac{\phi^n}{{M_p}^n}$ contain the factors $\frac{V(\phi)}{{M_p}^4}$ and $\frac{m^2(\phi)}{{M_p}^2}$ [2]. Consequently, quantum gravity corrections to $V(\phi)$ become large not at $\phi > M_p$, as one could infer from (26), but only at super-Planckian energy density, or for super-Planckian masses.

For example, quantum gravity corrections to the effective potential in the simplest chaotic inflation model with $V(\phi) = \frac{m^2}{2}\phi^2$ contain such terms as $m^2\phi^2 \times \frac{m^2}{M_p^2}$ and $m^2\phi^2 \times \frac{m^2\phi^2}{M_p^4}$. Only the last term could be dangerous, but not until $V(\phi)$ becomes greater than the Planck energy density. This justifies chaotic inflation models in the context of the simplest models of scalar field coupled to gravity [2].

One way to understand our main argument is to consider the case where the potential of the field ϕ is a constant, $V = V_0$, and the field ϕ does not give masses to any fields. Then the theory has a *shift symmetry*, $\phi \to \phi + c$. This symmetry is not broken by perturbative quantum gravity corrections, so no such terms as $\sum_n \lambda_n \frac{\phi^{4+n}}{{M_p}^n}$ are generated. This symmetry might be broken by nonperturbative quantum gravity effects (wormholes? virtual black holes?), but such effects, even if they exist, can be made exponentially small [29].

However, in some theories the scalar field ϕ itself may have physical (geometric) meaning, which may constrain the possible values of the fields during inflation. The most important example is given by $N = 1$ supergravity. The effective potential of the complex scalar field Φ in supergravity is given by the well-known expression (in units $M_p = 1$):

$$V = e^K \left[K^{-1}_{\Phi\bar\Phi}\, |D_\Phi W|^2 - 3|W|^2\right]. \tag{27}$$

Here $W(\Phi)$ is the superpotential, Φ denotes the scalar component of the superfield Φ; $D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$. The kinetic term of the scalar field is given by $K_{\Phi\bar\Phi}\,\partial_\mu\Phi\partial_\mu\bar\Phi$. The standard textbook choice of the Kähler potential corresponding to the canonically normalized fields Φ and $\bar\Phi$ is $K = \Phi\bar\Phi$, so that $K_{\Phi\bar\Phi} = 1$. This immediately reveals a problem: At $\Phi > 1$ the potential is extremely steep. It blows up as $e^{|\Phi|^2}$, which makes it very difficult to realize chaotic inflation in supergravity at $\phi \equiv \sqrt{2}|\Phi| > 1$.[2]

[2] Note, however, that this issue is not specific to the super-

It took almost 20 years to find a natural realization of chaotic inflation model in supergravity. Kawasaki, Yamaguchi and Yanagida suggested to take the Kähler potential

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + X\bar{X} \tag{28}$$

of the fields Φ and X, with the superpotential $m\Phi X$ [30].

At the first glance, this Kähler potential may seem somewhat unusual. However, it can be obtained from the standard Kähler potential $K = \Phi\bar{\Phi} + X\bar{X}$ by adding terms $\Phi^2/2 + \bar{\Phi}^2/2$, which do not give any contribution to the kinetic term of the scalar fields $K_{\Phi\bar{\Phi}}\, \partial_\mu\Phi\partial_\mu\bar{\Phi}$. In other words, the new Kähler potential, just as the old one, leads to canonical kinetic terms for the fields Φ and X, so it is as simple and legitimate as the standard textbook Kähler potential. However, instead of the U(1) symmetry with respect to rotation of the field Φ in the complex plane, the new Kähler potential has a *shift symmetry*; it does not depend on the imaginary part of the field Φ. The shift symmetry is broken only by the superpotential.

This leads to a profound change of the potential (27): the dangerous term e^K continues growing exponentially in the directions $(\Phi+\bar{\Phi})$ and $|X|$, but it remains constant in the direction $(\Phi - \bar{\Phi})$ (shift symmetry). Decomposing the complex field Φ into two real scalar fields, $\Phi = \frac{1}{\sqrt{2}}(\eta + i\phi)$, one can find the resulting potential $V(\phi, \eta, X)$ for $\eta, |X| \ll 1$:

$$V = \frac{m^2}{2}\left(\phi^2(1+\eta^2) + |X|^2 + \frac{3}{2}\eta^2\right). \tag{29}$$

This potential has a deep valley, with a minimum at $\eta = X = 0$. Therefore the fields η and X rapidly fall down towards $\eta = X = 0$, after which the potential for the field ϕ becomes $V = \frac{m^2}{2}\phi^2$. One can show that the inflaton potential $V = \frac{m^2}{2}\phi^2$ is not modified by radiative corrections for $V \ll M_p^4$. This provides a very simple realization of chaotic inflation in supergravity [30], and a counterexample to the arguments of Refs. [15, 26].

Note that there are several other ways to achieve inflation in supergravity, such as F-term and D-term hybrid inflation models [31, 32]. However, in our opinion, the model of chaotic inflation proposed in [30] is by far the simplest and the most natural inflationary model based on supergravity. It is especially interesting therefore that tensor perturbations predicted by this model have a relatively large amplitude. Other models of chaotic inflation in supergravity based on the idea of shift symmetry can be found in [33, 34].

Planckian values of the fields, as it leads to problems in realizing inflation in supergravity even for $|\Phi| \ll 1$. Thus this problem is not directly linked to detectability of tensor modes.

In string theory the situation is more complicated. A thorough investigation of inflation in string theory became possible only very recently, after the discovery of the KKLT mechanism of stabilization of extra dimensions [35]. As a result, only few examples of string theory inflation are presently available. Most of these models are based on various versions of hybrid inflation, and the amplitude of tensor perturbations there typically is very small, see e.g. [36, 37].

There is a family of string theory motivated models where inflation is supposed to occur due to rolling in the axionic direction [38, 42], as in the 'natural inflation' scenario [39]. One should note that the combined potential of the axion field and the volume modulus in the KKLT scenario is very much different from the effective potential of 'natural inflation'. In particular, the volume modulus is usually destabilized near the maximum of the axion potential, where one would naively expect to have a better chance of development of inflation. The only fully analyzed model of this type leads to a very small magnitude of tensor perturbations [40]. This model belongs to the general class of the moduli inflation models in string theory, as opposite to brane inflation [36, 37]. A recent version of the moduli inflation based on a modified KKLT mechanism [41] also leads to small tensor perturbations.

However, it was argued in [42] that if one considers simultaneous evolution of many types of axion fields, inflation may occur in a vicinity of the minimum of the axion potential, where the potential is quadratic, as in the simplest chaotic inflation scenario. This again leads to a large amplitude of tensor perturbations. There are some other string inflation models where tensor perturbations could be quite large if one takes into account nontrivial kinetic terms for the inflaton field [43].

One of the problems with all of these scenarios is that the large energy density, which could drive inflation and produce large tensor perturbations, tends to destabilize vacuum and decompactify extra dimensions, making our universe ten-dimensional [44]. According to [44], there is a general upper bound on the Hubble constant in all inflationary models based on the original version of the KKLT scenario: $H \lesssim m_{3/2}$, where $m_{3/2}$ is the gravitino mass. In this case the amplitude of tensor perturbations will be

$$\delta_T \sim \frac{H}{M_p} \lesssim \frac{m_{3/2}}{M_p}\ . \tag{30}$$

If one substitutes there the often discussed value $m_{3/2} \sim 1$ TeV $\sim 10^{-15}M_p$, one finds an extremely small amplitude of tensor perturbations $\delta_T \lesssim 10^{-15}$, which corresponds to the tensor-to-scalar ratio $r \ll 10^{-20}$. This is far below the most optimistic bounds for detectability of tensor modes [28, 45, 46].

However, recently a new generation of phenomenological models was proposed, where the gravitino mass can be very large, see e.g. [47, 48, 49]. Also, in a modified version of the KKLT scenario proposed in [44] one

can have inflation with the Hubble constant much higher than the gravitino mass. This may allow stringy inflation with a large amplitude of tensor perturbations.

IV. CONCLUSIONS

In this paper we analyzed the possibility to suppress the amplitude of adiabatic perturbations produced during inflation. We considered two possible mechanisms of such suppression: damping of initial perturbations by the late-time entropy release [11, 12] and cancellation of initial adiabatic perturbations by adiabatic perturbations produced, e.g., by the curvaton mechanism [21, 22, 23, 24, 25]. We conclude that neither of these mechanisms can lead to suppression of the initial adiabatic perturbations.

We pointed out, however, that there are no firm theoretical arguments against detectability of tensor modes in inflationary theory. If tensor modes are detected by the future CMB polarization experiments, it will be a great triumph and a powerful evidence in favor of the simplest versions of chaotic inflation. On the other hand, in most of the other inflationary models the amplitude of tensor perturbations is undetectably small, and it cannot be boosted up by damping the amplitude of adiabatic perturbations and increasing the tensor-to-scalar ratio. Therefore in planning the future CMB polarization experiments one should keep in mind that if the amplitude of tensor modes happens to be too small, it will rule out the simplest versions of chaotic inflation, but it may not tell us much about many other inflationary models.

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